

Andre Joyal on Myles Tierney*

October 10, 2017

Dear All,

As you may know, my friend and collaborator Myles Tierney passed away a few days ago. I thank you all for the condolences expressed. I would like to briefly recall the contributions of Myles to mathematics and our collaboration.

Myles received his Phd from Columbia University in 1965, under the direction of Samuel Eilenberg with a dissertation on "Classifying Spaces for K -Theory mod P ". He belongs to a cohort of Eilenberg's students (Lawvere:1963, Applegate:1965, Tierney:1965, Beck:1967) that was to have a major influence on the development of category theory. After his Phd, he was invited by Eldon Dyer to join the faculty at Rice university in Houston. He left Rice after 15 months to visit the ETH Forschungsinstitut für Mathematics in Zurich, where Beno Eckmann was assembling a group of category theorists and algebraic geometers that included Bill Lawvere (1964-67,1968-69), Myles Tierney (1966-1968), Peter Gabriel, Jon Beck, Peter Freyd and Mike Barr. Myles worked on the simplicial construction of spectra and he published his work in a Lecture Notes "Categorical Constructions in Stable Homotopy Theory". Lawvere worked on the axiomatic of the category of categories and he collaborated with Myles on finding new axioms for toposes. Having introduced the sub-object classifier, he discovered the notion of elementary topos and Myles discovered that a Grothendieck topology is the same thing as a closure operator on the sub-object classifier. They kept developing the theory of elementary topoi when they both joined Dalhousie University in 1969. While at Dalhousie, they organised an international conference on topos theory (January 1971) with the participation of algebraic geometers, category theorists and logicians (Bucur, Giraud, Goodman, Kock, Myhill, Illusie, Lambek, Scott, Verdier, Tierney, etc). The proceedings of the conference include the paper "Sheaf theory and the Continuum Hypothesis" in which Myles constructs a boolean topos satisfying the axiom of choice but for which the Continuum Hypothesis fails. His construction showed that the

*Here is the obituary that Andre Joyal wrote after death of his long-term collaborator Myles Tierney on category theory mailing list. All credit is due to Joyal. This document is only meant to store this well-written and touching obituary since I could not find a link to it on category mailing list website and I copied it from my email inbox. Nothing has been added or retracted from Original email sent by Joyal.

forcing methods introduced by Paul Cohen were essentially sheaf theoretic in nature. The paper had a strong influence on the subsequent developments of the logical aspects of topos theory. Unfortunately, the collaboration between Myles and Bill ended in Dalhousie, partly because of the political turmoil of that period. Myles joined the faculty at Rutgers and Bill the faculty at the University of Buffalo NY. Myles has two papers on classifying topoi; in the first, "On the spectrum of a Ringed Topos", he uses the Stone dual of the spectrum of a commutative ring to give a new construction of Hakim's spectrum of a ringed topos; in the second paper "Forcing topologies and Classifying Topoi", he shows that any (elementary) geometric theory has a classifying (elementary) topos. The two papers were published in the proceedings of a conference in honour of Eilenberg, with the participations of Maurice Auslander, Kuo-Tsai Chen, Barry Mitchell, Eldon Dyer, Peter Freyd, John Gray, Dale Husemoller, Daniel Kan, Bill Lawvere, Saunders Mac Lane, John Moore and Alex Rosenberg. I began to collaborate with Myles in 1979, but our work was brutally interrupted by a terrible accident that left Myles physically crippled for the rest of his life. His determination to survive and his willingness to keep doing mathematics were admirable. We eventually published the fruit of our collaboration in the AMS Memoir "An extension of the Galois theory of Grothendieck". We showed that every Grothendieck topos can be represented by a localic groupoid. The result was shedding some light on Grothendieck's view that toposes are generalised spaces. Grothendieck had introduced various cohomology theories for toposes (we may argue that toposes were precisely invented for that reason) but there was virtually no homotopy theory. We wondered if one could construct the analog of a Postnikov tower for a topos? There was a clear analogy between atomic toposes (introduced by Barr) and $K(G, 1)$ spaces. I visited Columbia University in 1982-84 and we had a weekly seminar with the participation of Alex Heller and Bob Thomason. During that time, I succeeded in constructing a Quillen model structure on the category of simplicial objects of any Grothendieck topos; the model structure has applications to higher stacks and to non-abelian cohomology, but the connection between the topos structure and the model structure was mysterious. We began to investigate this question systematically. We eventually showed that the category of internal groupoids in the topos of simplicial sets admits a model structure which is Quillen equivalent to the model category of simplicial sets. We later extended the equivalence to the category of simplicial objects in a general Grothendieck topos.

In retrospect, the hypothesis that the topos of simplicial sets was the right framework for understanding the homotopy theory of spaces was off the mark. A better framework is the notion of *model topos* (=higher topos) introduced by Charles Rezk in the late nineties and which served as a framework in the derived algebraic of Bertrand Toën, Gabriele Vezzosi and Jacob Lurie. But we were not entirely off the mark! The notion of higher topos depends on higher categories to the same extent that the notion of topos depends on categories (and the notion of Heyting algebra depends on posets). In our paper "Quasi-categories vs Segal spaces", we show that the model structure for quasi-categories is Quillen equivalent to the model category for complete Segal spaces (introduced by Rezk) and to the model category

for Segal categories (introduced by Hirschowitz and Simpson). The equivalences now fit into a larger network described by Julie Bergner and which provides a foundation to higher category theory.

Myles was the advisor of seven phd:

Ira Wolf (Rutgers, 1971) Radu Diaconescu (Dalhousie, 1973) Carol Keller (Rutgers, 1983) Norman S. Adam (Rutgers, 1984) Terence Lindgren (Rutgers, 1984) Todd Trimble (Rutgers, 1994) Mauri Luca (Rutgers, 1998)

Radu Diaconescu has many contributions to topos theory. Tod Trimble has proved an important coherence theorem for star-autonomous categories; he has contributions to higher category theory.

For many years, Myles organised with Heller a yearly NY category theory meeting. Hanne and Myles joyfully opened their Soho loft to all. I often slept on the floor in a sleeping bag! Nobody wanted to miss the saturday night gathering which often included Eilenberg, Mac Lane, Heller, Linton, Bass, Lawvere, Beck, Freyd, Barr, Mitchell, Dubuc, Duskin, Paré, Makkai, Bunge, Diaconescu, Pelletier, Niefield, Schanuel, MacDonald, Wood, Rosebrugh, Tholen, Scedrov, Blass, Gray and many others, sometime visitors from Europe and Australia. Thank you Hanne and Myles!

André