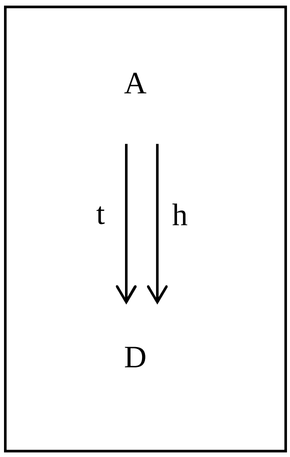
A Map in the Category of Graphs

Let’s look at the category of graphs, which has graphs such as O shown below as objects.

O

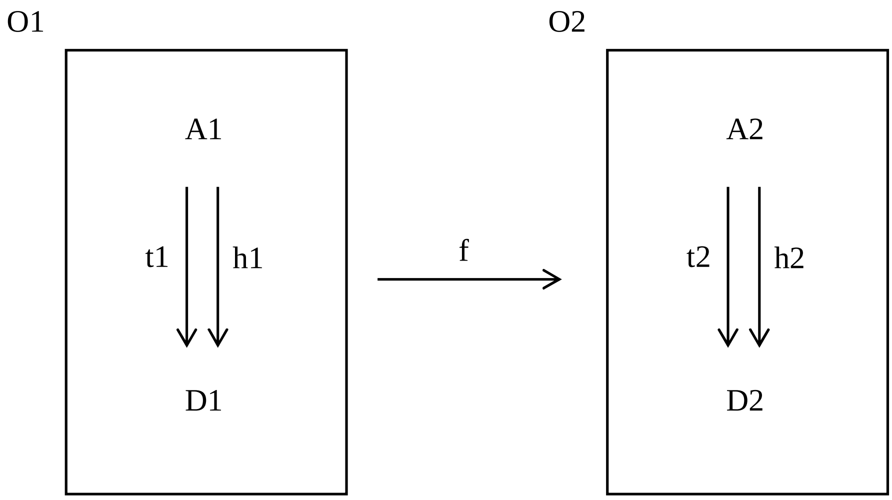


Object O is a pair of sets: set A of arrows, set D of dots; and a parallel pair of functions: function t assigns to each arrow (in A) its source dot (tail; in D); function h assigns to each arrow (in A) its target dot (head; in D).

A map f from an object O1 to an object O2

f: O1 --> O2

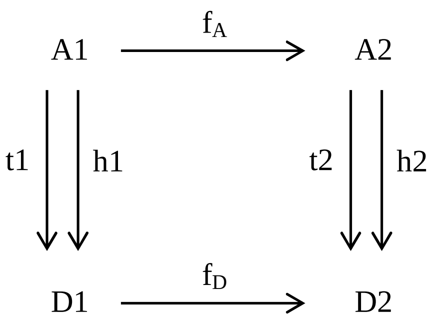
depicted as



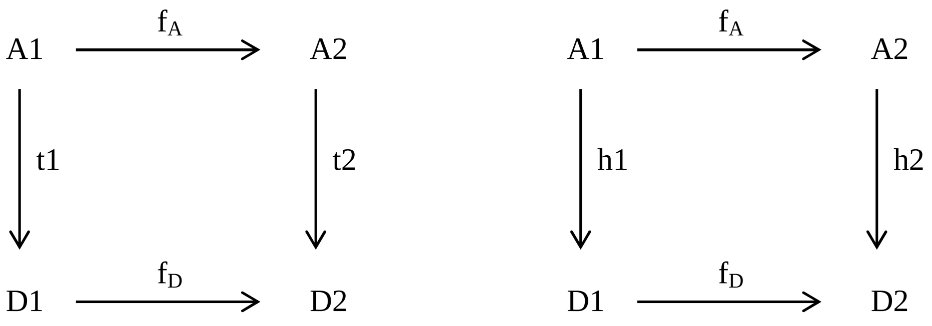
is a pair of functions

$f = \langle f_A, f_D \rangle$

depicted as



or as a pair of commutative squares



satisfying

$t_2 f_A = f_D t_1$

corresponding to the square on the left and

$h_2 f_A = f_D h_1$

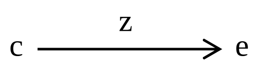
corresponding to the square on the right (in the above).

Let’s consider a map

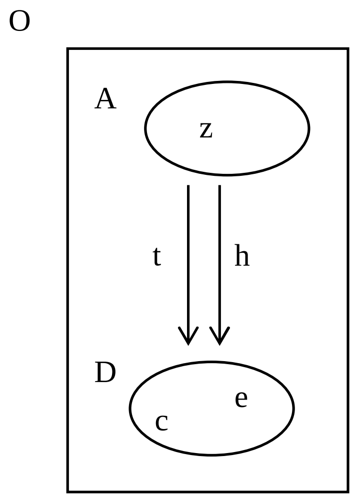
f: O --> O

from domain object O to codomain object O to illustrate the idea of map in the category of graphs in some more detail.

Let’s take a graph



as our object O



A = {z} and D = {c, e} are the pair of sets of arrows and dots, respectively of object O.

t: A --> D, with t(z) = c

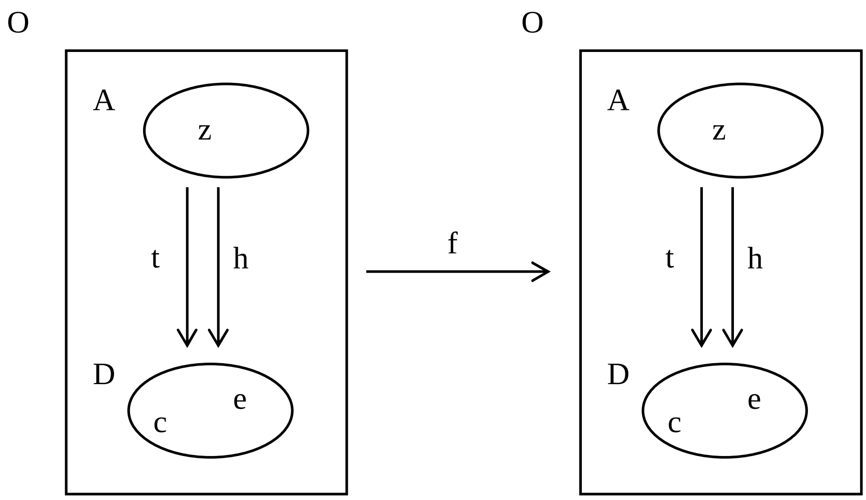
and

h: A --> D, with h(z) = e

are the parallel pair of functions of tail (source) and head (target), respectively of the object O.

Now, the map

f: O --> O



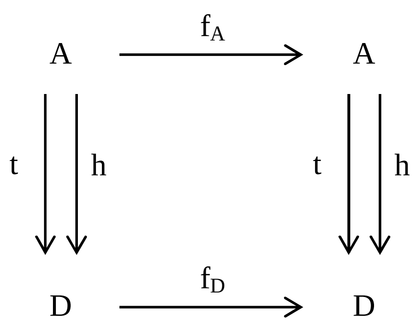
which we recollect as

$f = <f_A, f_D>$

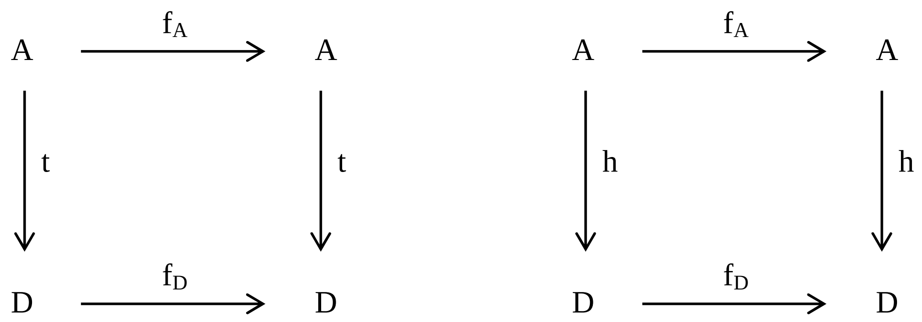
with

f_A: A --> A and f_D: D --> D

depicted as



and after separating heads from tails



satisfies

$tf_A = f_Dt$ and $hf_A = f_Dh$

Now, we have a question!

What does ‘a map $f: O \dashrightarrow O$ is a pair of functions $f = <f_A, f_D>$ satisfying $tf_A = f_Dt$ and $hf_A = f_Dh$’ mean?

What do we have here? We have 4 functions:

t: A --> D

h: A --> D

f_A: A --> A

f_D: D --> D

of which we already know, clearly, what the functions tail t and head h are. But first, let’s write the domain and codomain sets of the functions.

A = {z}

D = {c, e}

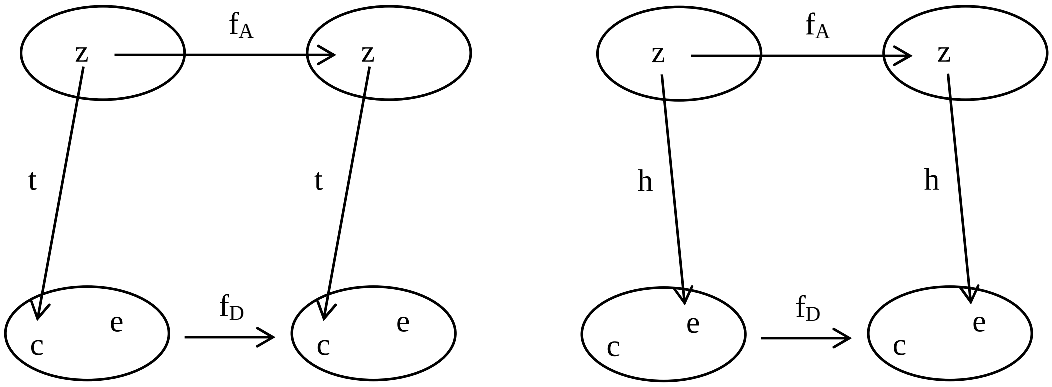
The function t: A --> D is given by t(z) = c, and the function h: A --> D is given by h(z) = e.

How about the functions f_A and f_D?

Let's start with f_A: A --> A.

Since A = {z}, there is only one possibility for f_A; the function f_A assigns the only element z of the codomain set A to the one element z of the domain set A; $f_A(z) = z$.

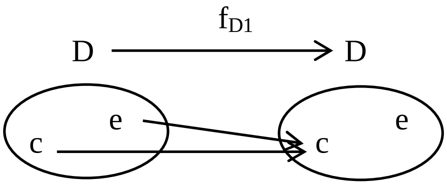
Before we go on to f_D: D --> D, let's depict diagrammatically all that we stated above as

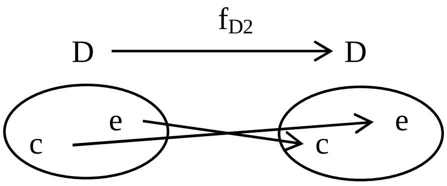


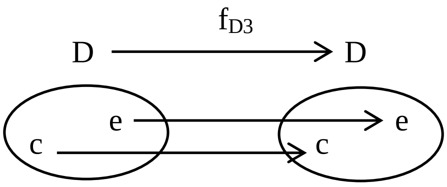
Now in order for the f: O --> O to be a map, function f_D: D --> D, D = {c, e} must satisfy $tf_A = f_Dt$ and $hf_A = f_Dh$.

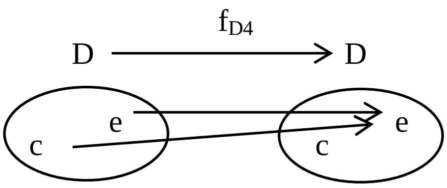
What is function f_D? f_D is a function f_D: D --> D from domain set D = {c, e} to codomain set D = {c, e}.

Given that there are 2 elements in the domain set D and 2 elements in the codomain set D, we have a total of 4 (2^2) functions from D to D as shown below:









Now in order to find out how many maps there are from the object O to O, we have to see how many of the following 4 pairs of equations hold true.

1. $tf_A = f_{D1}t$ and $hf_A = f_{D1}h$

2. $tf_A = f_{D2}t$ and $hf_A = f_{D2}h$

3. $tf_A = f_{D3}t$ and $hf_A = f_{D3}h$

4. $tf_A = f_{D4}t$ and $hf_A = f_{D4}h$

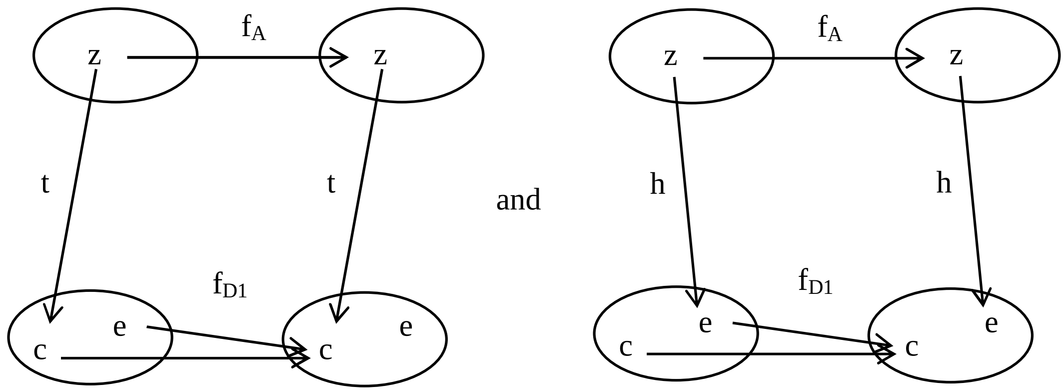
Restating what we just said, we say

$f_1 = \langle f_A, f_{D1}\rangle: O \to O$

is a map from domain object O to codomain object O if

$tf_A = f_{D1}t$ and $hf_A = f_{D1}h$

or pictorially, if



commute.

We say a diagram, for example, the square on the left commutes if $tf_A = f_{D1}t$.

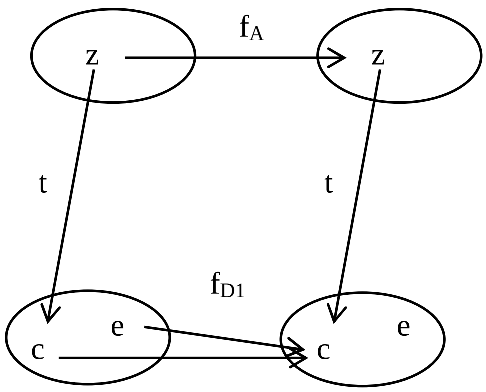
OK, fine, but first let's list out all 3 functions in the equation $tf_A = f_{D1}t$ to be satisfied:

$t(z) = c$

$f_A(z) = z$

$f_{D1}(c) = c$ and $f_{D1}(e) = c$

which is what is depicted in the diagram



Let's take off at the top-left z; we can take f_A and go to z, and from z take t to land at c.

Or, we can take t, from the very same top-left z, and go to c, and from c take f_{D1} to land at c. Both itineraries take us from z at the top-left to the very same down-right c. Speaking less verbally, we evaluate both the left-hand side and the left-hand side of the equation

$tf_A = f_{D1}t$

at z to see if the equation

$tf_A = f_{D1}t$

holds true.

Left-hand side

$tf_A(z) = t(z) = c$

Right-hand side

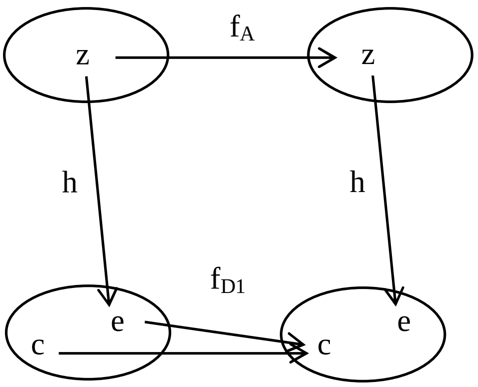
$f_{D1}t(z) = f_{D1}(c) = c$

Therefore

$tf_A = f_{D1}t$

which is not surprising given that we already saw that the corresponding diagram commutes.

Now let's see if our diagram on the right (above) corresponding to heads



commutes, for which we check if $hf_A = f_{D1}h$.

Evaluating at z

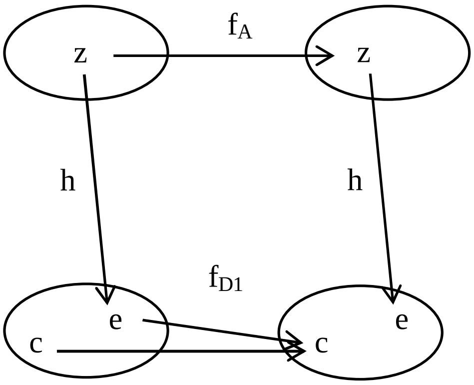
$hf_A(z) = h(z) = e$

$f_{D1}h(z) = f_{D1}(e) = c$

we find that

$hf_A \neq f_{D1}h$

i.e.



doesn't commute.

Let's remind ourselves what we are doing now. We started out saying

$f_1 = \langle f_A, f_{D1} \rangle: O \dashrightarrow O$

is a map if

$tf_A = f_{D1}t$ and $hf_A = f_{D1}h$.

We found out that

$tf_A = f_{D1}t$

but

$hf_A \neq f_{D1}h$.

So $f_1 = \langle f_A, f_{D1} \rangle$ is not a map.

How about

$f_2 = \langle f_A, f_{D2} \rangle$

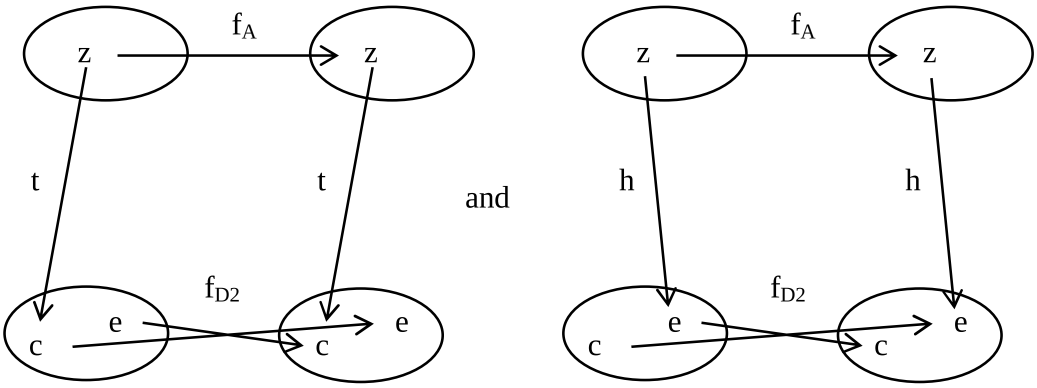
$f_3 = \langle f_A, f_{D3} \rangle$

$f_4 = \langle f_A, f_{D4} \rangle$

Let's look at

$f_2 = \langle f_A, f_{D2} \rangle$

f_2 is a map if



commute.

In terms of equations,

if $tf_A = f_{D2}t$ and $hf_A = f_{D2}h$,

then f2 = <fA, fD2> is a map.

Let's first look at the equation on the left

$tf_A = f_{D2}t$

and evaluate both sides of the equation at z.

$tf_A(z) = t(z) = c$

$f_{D2}t(z) = f_{D2}(c) = e$

Therefore, $tf_A \neq f_{D2}t$.

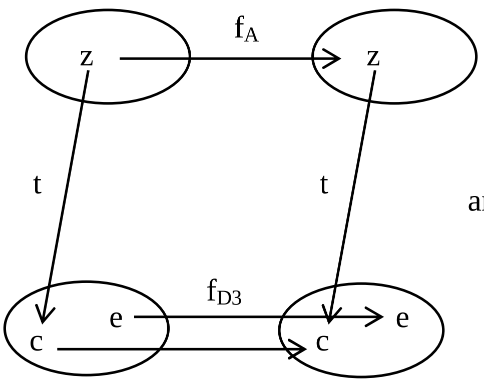
Since we need both equations

$tf_A = f_{D2}t$ and $hf_A = f_{D2}h$

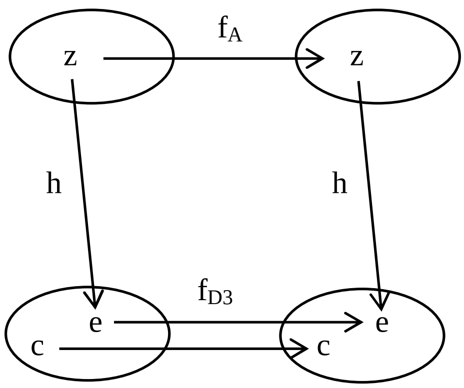
to hold true for $f_2 = <f_A, f_{D2}>$ to be a map, and since we found $tf_A \neq f_{D2}t$, we won't bother checking the other equation, and conclude $f_2 = <f_A, f_{D2}>$ is not a map.

How about $f_3 = <f_A, f_{D3}>$?

Does the pair of diagrams



and



commute?

We have to check if

$tf_A = f_{D3}t$ and $hf_A = f_{D3}h$

which we can also do by following the arrows in the diagram in addition to substituting symbols in the equations.

Evaluating both sides of the equation on the left at z

$tf_A(z) = t(z) = c$

$f_{D3}t(z) = f_{D3}(c) = c$

Therefore, the equation $tf_A = f_{D3}t$ holds true i.e. the corresponding diagram on the left commutes.

Next, evaluating $hf_A = f_{D3}h$ at z, we find that

$hf_A(z) = h(z) = e$

$f_{D3}h(z) = f_{D3}(e) = e$

Therefore, the equation $hf_A = f_{D3}h$ holds true i.e. the corresponding diagram on the right commutes.

Since

$tf_A = f_{D3}t$ and $hf_A = f_{D3}h$

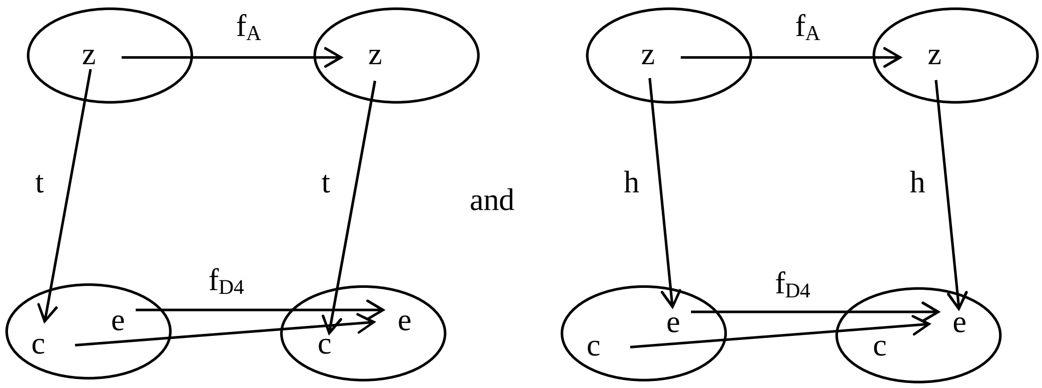
we say

$f_3 = \langle f_A, f_{D3} \rangle$

is a map f_3: O --> O from domain object O to codomain object O.

How about $f_4 = <f_A, f_{D4}>$?

Does the pair of diagrams



commute?

Is $tf_A = f_{D4}t$ and $hf_A = f_{D4}h$?

Evaluating both sides of the equation on the left at z, we find that

$tf_A(z) = t(z) = c$

$f_{D4}t(z) = f_{D4}(c) = e$

Therefore, $tf_A \neq f_{D4}t$. Thus, $f_4 = <f_A, f_{D4}>$ is not a map.

To sum up, of all the 4 possibilities

$f_1 = <f_A, f_{D1}>$

$f_2 = <f_A, f_{D2}>$

$f_3 = <f_A, f_{D3}>$

$f_4 = <f_A, f_{D4}>$

we found that only

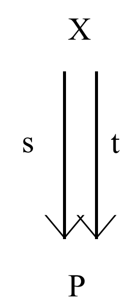
$f_3 = <f_A, f_{D3}>$ is a map f_3: O --> O from the domain object O to the codomain object O.

Well, what does all this mean? Where's the big-picture? Here, it might help to note that f_{D1} mapped both head and tail to tail, f_{D2} mapped tail to head and head to tail, and f_{D4} mapped both tail and head to head, while f_{D3} mapped head to head and tail to tail.

So?

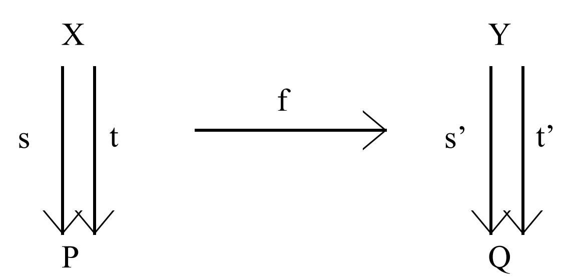
Composition of Maps in the Category of Graphs

An object of the category of graphs is a parallel pair of functions

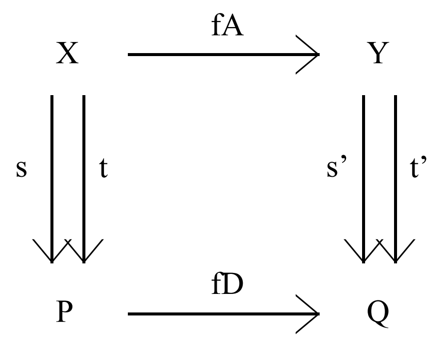


where X is called the set of arrows and P the set of dots of the graph. If x is an arrow (element of X), then s(x) is called the source of x, and t(x) is called the target of x.

A map

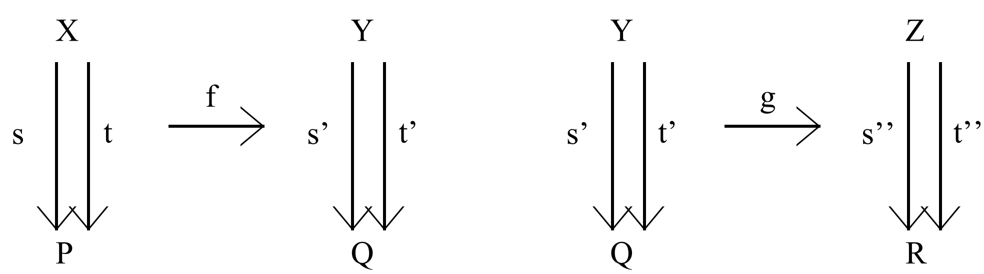


in the category of graphs is defined to be any pair of functions f_A: X --> Y, f_D: P --> Q for which the diagram

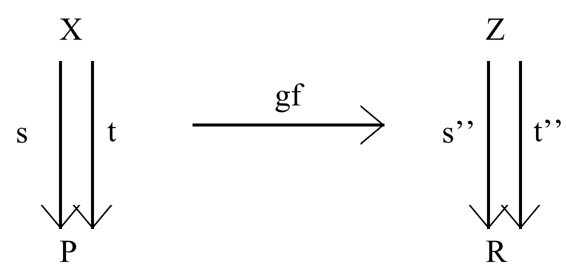


commutes satisfying $f_D s = s' f_A$ and $f_D t = t' f_A$.

What is the composite map gf of the maps f and g depicted below



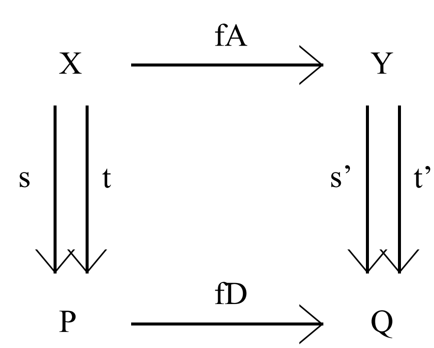
The composite map gf of maps f and g is



Is the above composite map gf a map in the category of graphs?

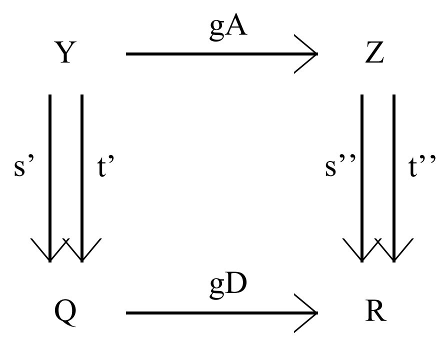
First, let's look at the maps f, g in the category of graphs of which gf is composite.

The map f, when spelled-out, is



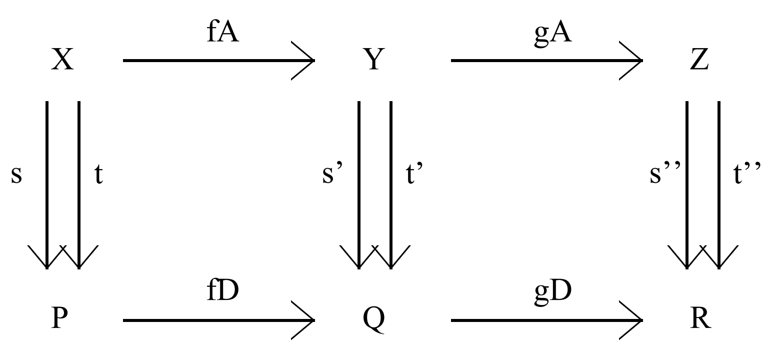
satisfying $f_D s = s' f_A$ and $f_D t = t' f_A$

The map g, when spelled-out, is

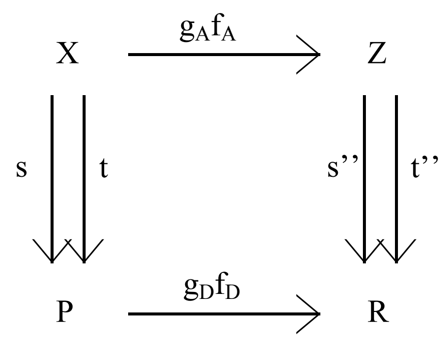


satisfying $g_D s' = s'' g_A$ and $g_D t' = t'' g_A$

The composite gf of maps g after f



which is equal to



which must satisfy

$s'' g_A f_A = g_D f_D s$ and $t'' g_A f_A = g_D f_D t$

for the composite gf to be a map in the category of graphs.

We know, going by the fact that f and g are maps in the category of graphs, that

$f_D s = s' f_A$ and $f_D t = t' f_A$

and

$g_D s' = s'' g_A$ and $g_D t' = t'' g_A$

and that we have to check to see if $s'' g_A f_A = g_D f_D s$ and $t'' g_A f_A = g_D f_D t$

$s'' g_A f_A = g_D s' f_A = g_D f_D s$ and $t'' g_A f_A = g_D t' f_A = g_D f_D t$

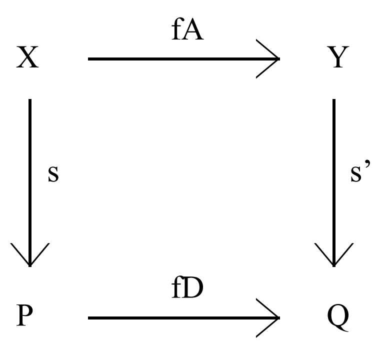
Therefore…; I'll let you conclude, but given that we, often, look at one thing and see something (plz don't press that panic button; I am saving my symbolic conscious experience for sometime later), what do we see when we look at symbol substitution in, say,

$s''g_A f_A = g_D s' f_A = g_D f_D s$

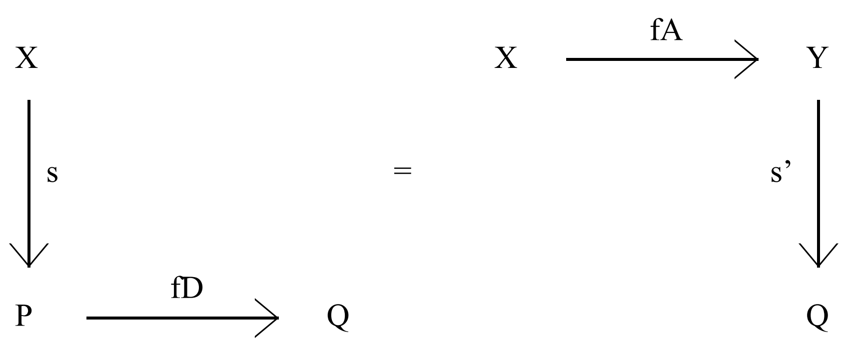
Given

$f_D s = s' f_A$

we look at



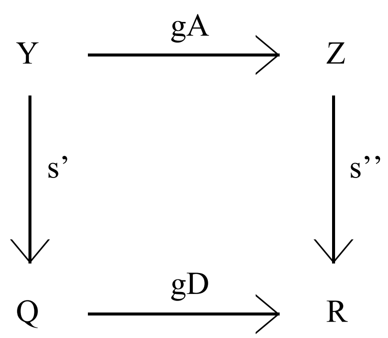
and see



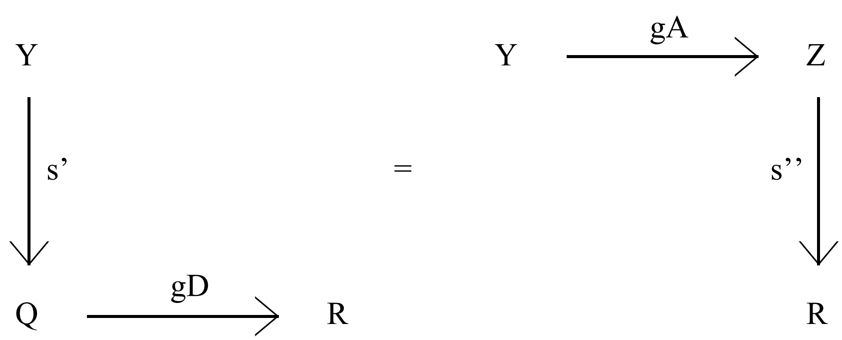
Again, given

$g_D s' = s'' g_A$

we look at



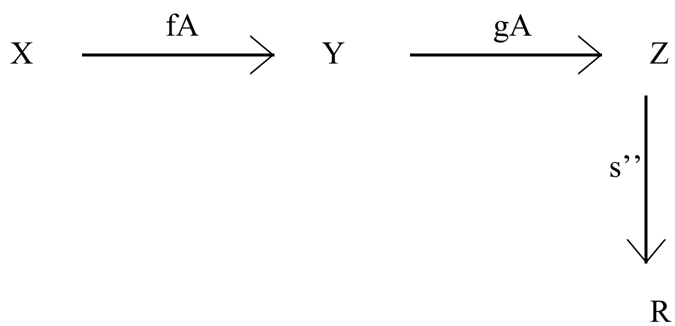
and, from our vantage point, see



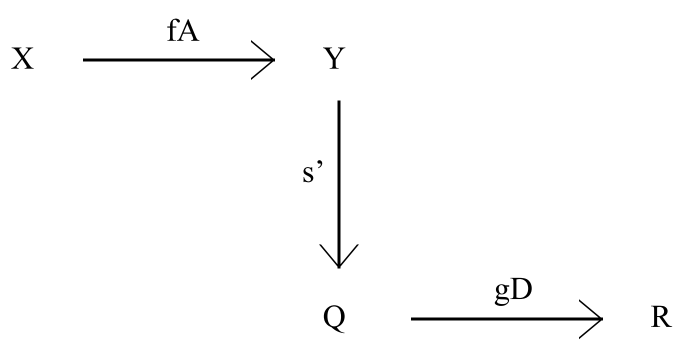
Now, from this perspective, when we look at

$s''g_Af_A = g_Ds'f_A = g_Df_Ds$

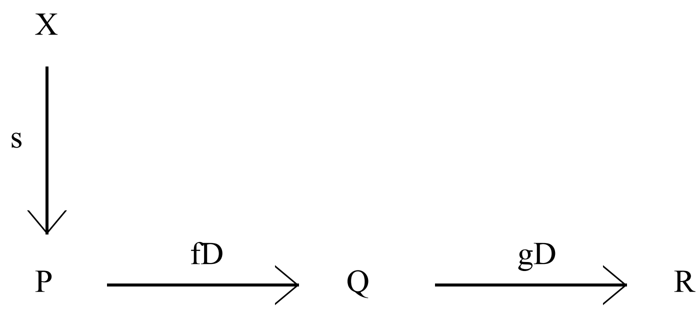
we see



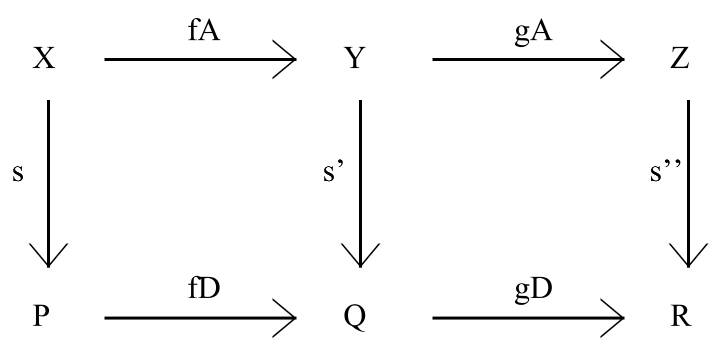
equal to



equal to



in



If you say so.

Now, what do we see when we look at

$t''g_Af_A = g_Dt'f_A = g_Df_Dt$

OK, fine. No more drops dripping on to forehead; for now let's just friend—facebook—

substitution and composition.

See you soon, alligator!

Identity Maps in the Category of Graphs

First, let's look at the definition of CATEGORY.

A category consists of the data:

(1) Objects A, B, C, …
(2) Maps f, g, h, …
(3) For each map f, one object A as domain of f and one object B as codomain of f as in f: A --> B.
(4) For each object A, an identity map with object A as both domain and codomain of the identity map as in 1_A: A --> A.
(5) For each composable pair of maps f: A --> B, g: B --> C with domain of g, B equal to codomain of f, B, a composite map gf with the domain of f, A as domain and the codomain of g, C as codomain as in gf: A --> C.

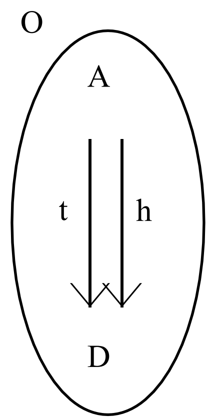
The above data of category satisfy the following rules:

(1) Identity laws: If f: A --> B, then $1_B f$ = f and $f1_A$ = f.
(2) Associative law: If f: A --> B, g: B --> C, and h: C --> D, then (hg)f = h(gf) = hgf.

Now that we have seen Category, let's look at Category of Graphs.

An object O of the category of graphs is a parallel pair of functions called tail, head with a set called arrows as domain and a set called dots as codomain of the pair of functions as

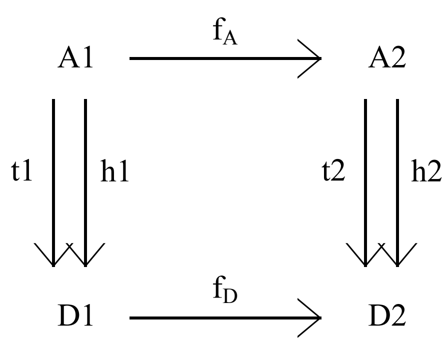
in t: A --> D, h: A --> D shown below:



A map f: O1 --> O2 from a domain object O1 (t1: A1 --> D1, h1: A1 -->D1) to a

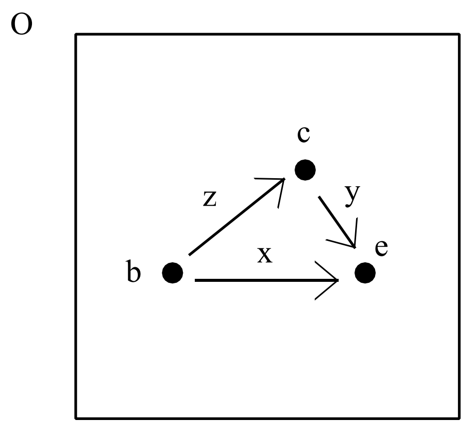
codomain object O2 (t2: A2 --> D2, h2: A2 --> D2) is a pair of functions f_A: A --> A,

f_D: D --> D as in



satisfying $t_2 f_A = f_D t_1$ and $h_2 f_A = f_D h_1$.

Before we go any further, let's look at an object O in the category of graphs

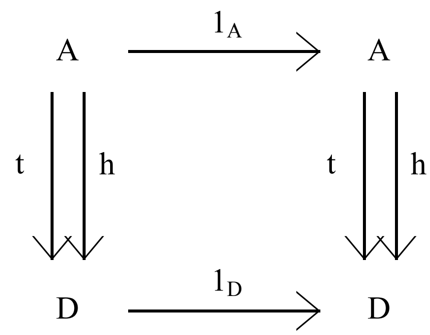


and save it to monkey later.

Now, if we look back at the definition of category, it looks like we recognized (1), (2), and (3) of the data of a category in the case of our category of graphs. Now we have to look for (4), i.e., identity map.

What's an identity map in the category of graphs? Thanks to the definition, we need not get lost in thought.

An identity map is a map. Before we unwrap this goodie, let's parrot the definition. For each object O (t: A --> D, h: A --> D), there is an identity map with object O as both domain and codomain of the identity map as in 1_O: O --> O. Now let's bite into the chocolate before it melts away. Let's recollect that a map (which is what an identity map is first and foremost) in the category of graphs is a pair of functions satisfying a pair of equations (our life couldn't have been easier), which when translated to the case of identity map 1_O: O --> O translates to a pair of identity functions 1_A: A --> A, 1_D: D --> D as in



satisfying a pair of equations $t1_A = 1_Dt$ and $h1_A = 1_Dh$.

Do they? Don't we have to check to see if $t1_A = 1_Dt$ and $h1_A = 1_Dh$? Aren't we defining? If so, by definition, isn't $t1_A = 1_Dt$ and $h1_A = 1_Dh$. Well, don't we want our definition of Category of Graphs to be consistent with our, again, definition of Category? Definition, in delimiting, description, changes—changes in practice—in the practice of describing. Holy cow! For now, as an exit-strategy, let's just say we aren't modern enough—enough to go post-modern, go [all-out] postal. Jeez!

Let's now check if $1_O = (1_A, 1_D)$ is a map in the category of graphs. In other words, let's check if $t1_A = 1_Dt$ and $h1_A = 1_Dh$, noting that t: A --> D, h: A --> D.

Looking back at the definition of the category, we see:

If f: A --> B, then $1_B f = f$ and $f1_A = f$.

So, given t: A --> D, $1_D t = t$ and $t1_A = t$. With another so in tow, we have $t1_A = 1_D t$.

In a similar vein, given h: A --> D, $1_D h = h$ and $h1_A = h$. Therefore, as earlier, $h1_A = 1_D h$.

Thus the identity map 1_O: O --> O defined as a pair of identity functions 1_A: A --> A,

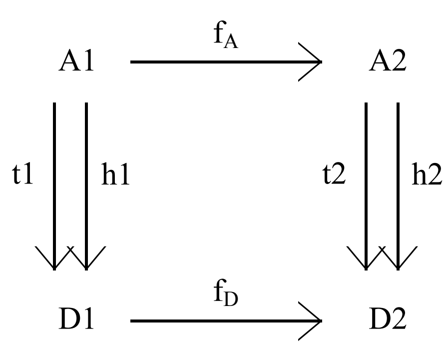
1_D: D --> D is indeed a map in the category of graphs.

Now, is the map 1_O: O --> O in the category of graphs an identity map in the category of graphs?

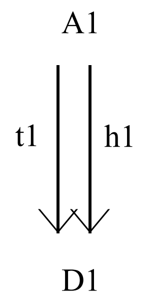
Wut!

Well, when in doubt, we study the definition—definition of category.

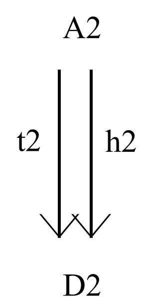
We have, in the category of graphs, a map



with a domain object

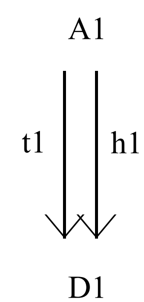


and a codomain object

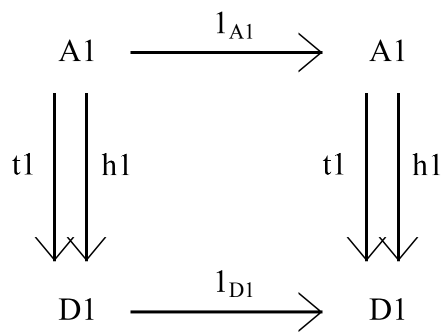


satisfying $t_2 f_A = f_D t_1$ and $h_2 f_A = f_D h_1$.

For each object of the category we have an identity map with the very object as both domain and codomain. So, corresponding to the object

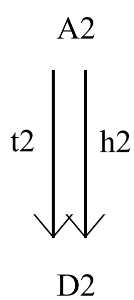


we have the identity map

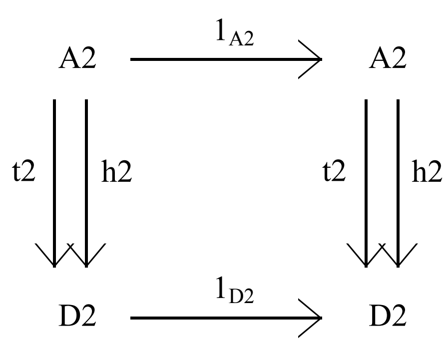


satisfying $t_1 1_{A1} = 1_{D1} t_1$ and $h_1 1_{A1} = 1_{D1} h_1$.

And corresponding to

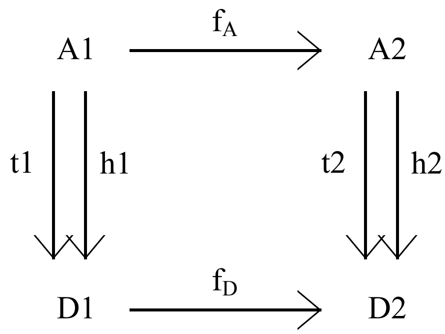


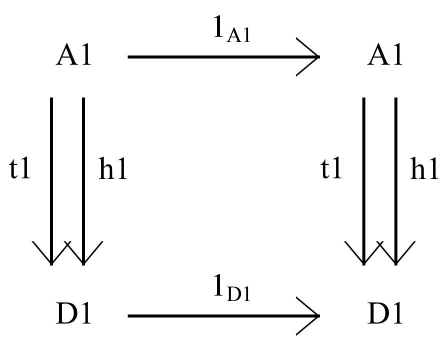
we have the identity map

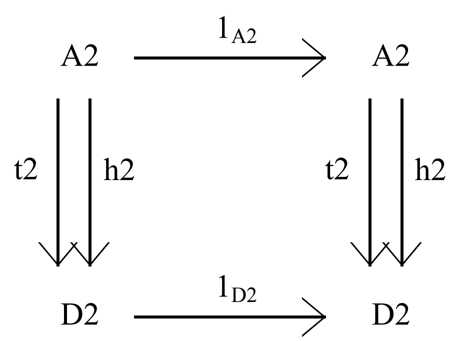


satisfying $t_2 1_{A2} = 1_{D2} t_2$ and $h_2 1_{A2} = 1_{D2} h_2$.

To sum up, we have three maps







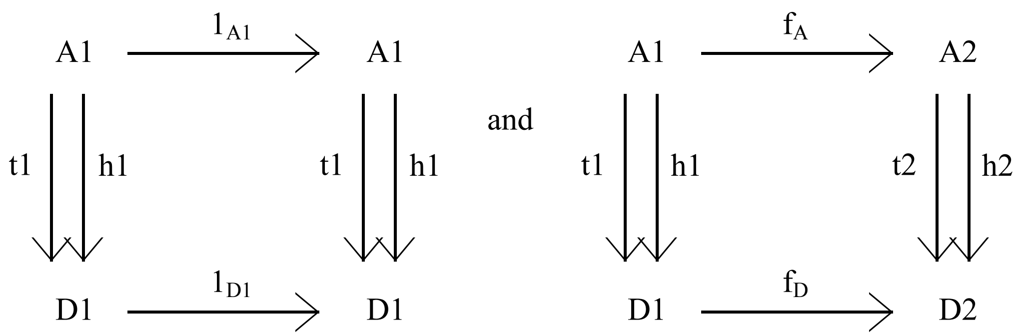
What are we going to do with this trinity?

Looking ahead, in the rear-view mirror, at the definition of category, we see the identity laws

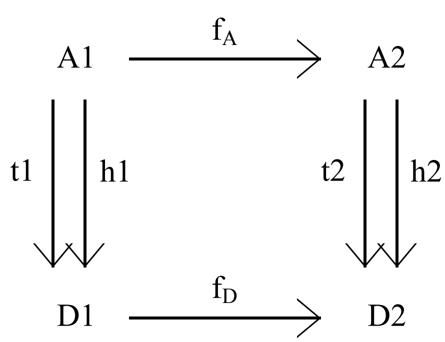
If f: A --> B, then $1_B f = f$ and $f1_A = f$

that maps in a category must satisfy.

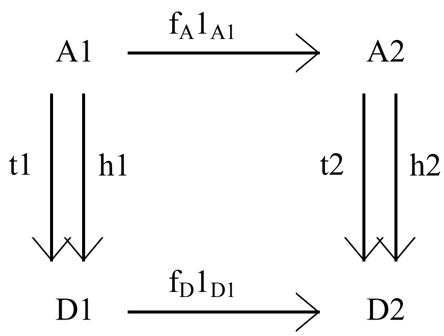
Importing these beautiful laws into our category of graphs, we see that we have to, first, see if the composite of



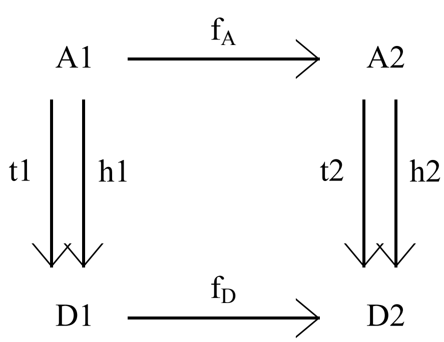
is equal to



The composite map of $(1_{A1}, 1_{D1})$ and (f_A, f_D) shown above can be drawn as

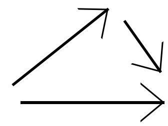


which is equal to



So is the case with the other identity law.

Now, I feel like, in explaining something, I said something like, ‘that’s what it means’ to which I can hear you say something like ‘what is that that that that it is supposed to mean?’ Or, in more politically-correct terminology, there’s always room for clarification, which I’ll provide in terms of the example



of an object in the category of graphs we saw earlier, but didn’t get a chance to look at.

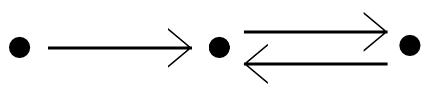
Isomorphisms in the Category of Graphs

Let’s do Exercise 6 (Conceptual Mathematics, page 159).

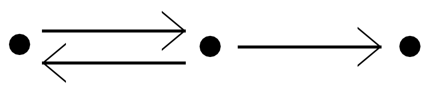
Exercise: Each of the following graphs is isomorphic to exactly one of the others.

Which?

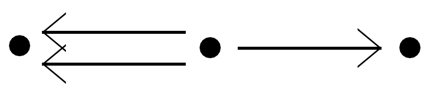
(1)



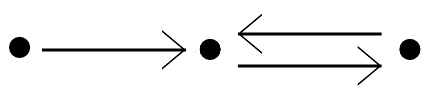
(2)



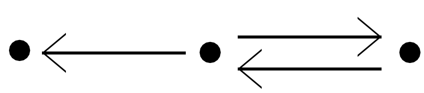
(3)



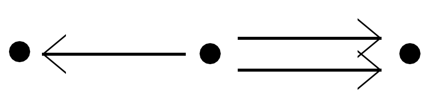
(4)



(5)



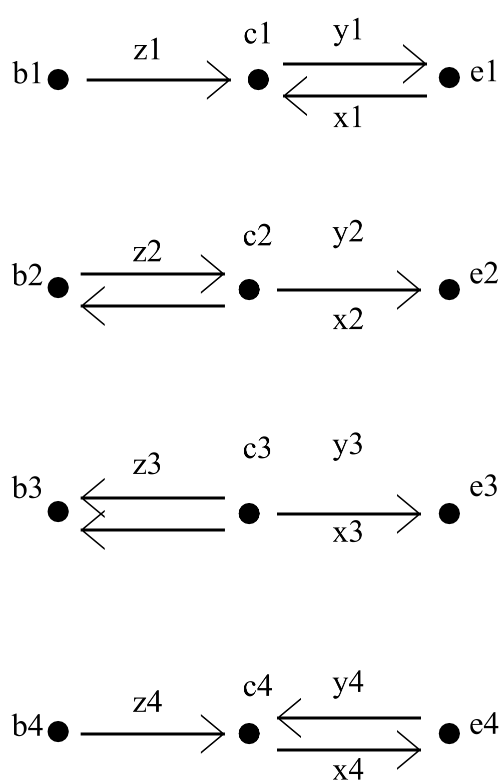
(6)

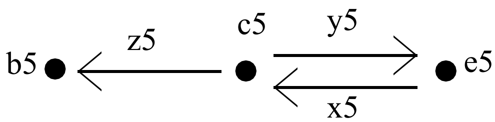


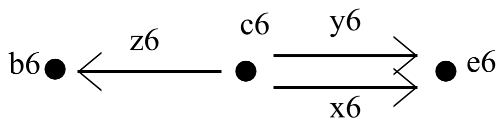
Earlier on (page 158) we learn that two graphs are isomorphic if we can exactly match arrows of one graph to arrows of the other and dots of one to dots of the other; in such a way that if two arrows are matched, then so are their source-dots and so are their target-dots. Listening to what we are just told we find it, comforting, notwithstanding the demanding exactness, to learn that math is our making—in our hands.

Whatever.

Let's first label the arrows and dots of the given six graphs.

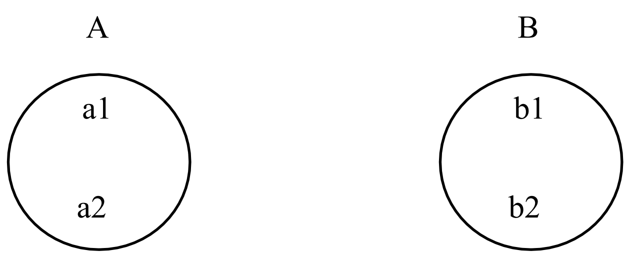






Now let's see how isomorphism looks like in the case of something much more familiar, say, sets.

Consider two sets A = {a1, a2} and B = {b1, b2} shown below:



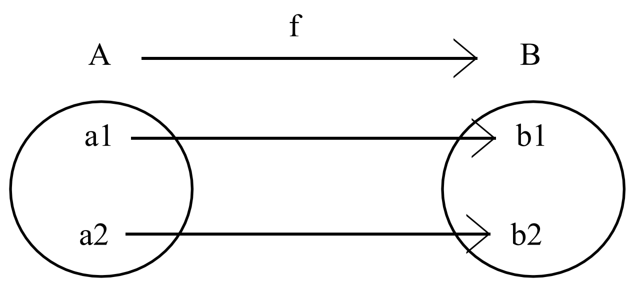
Clearly, A and B are two different sets. So, we can ask, 'are there any similarities between the two sets A, B?' In asking this question, we are rather boldly, but with good reason, asserting that two different things can be similar in more than one respect. (On a not so tangential note, when we are around kids, we often mistake their statements for questions—for not so well-formulated questions failing to recognize them as what they indeed are: answers we all knew full-well, but have forgotten during the course of our schooling by the society, which is what my sister's daughter Bhavana has been teaching me recently.). Well, this is not as high-funda as it sounds. After all, we can be similar in just one dimension, say, living, or in exactly two dimensions, say, living and feeling, so on and so forth.

Returning to our sets A and B, we say that A and B are isomorphic (same shape, which in the case of sets happens to be size) if there exists an isomorphism between A and B. Does this sound somewhat like: two different things are similar if there exits a similarity between the different things. It better; welcome to the wonderful wizard of obvious.

Now let's ask, 'what is isomorphism?' An isomorphism is a map. A function f: A --> B from domain set A to codomain set B is an isomorphism if there exits a function g: B --> A such that the composite function of f and g, gf: A --> B --> A = 1_A, the identity function on A and the composite function of g and f, fg: B --> A --> B = 1_B, the identity function on B.

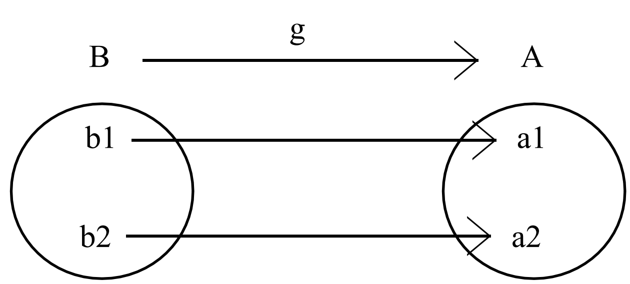
Given that we already know that the given two sets A = {a1, a2} and B = {b1, b2} are of the same size, i.e. |A| = |B| = 2, let's see if A and B are isomorphic. All we need is one isomorphism between A and B.

Consider a function f: A --> B, whose internal diagram is shown below:



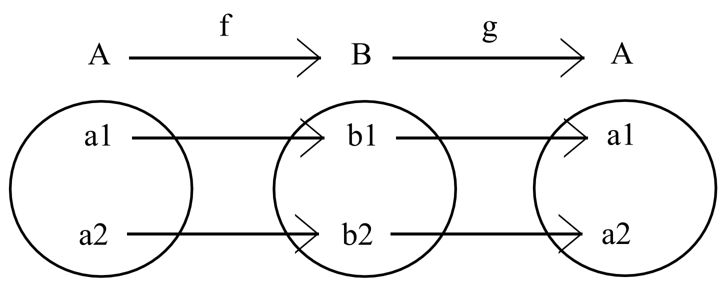
and in terms of equations f(a1) = b1 and f(a2) = b2

and a function g: B --> A, whose internal diagram is shown below:

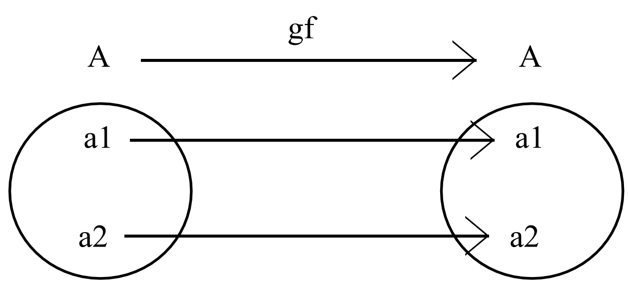


and in terms of equations g(b1) = a1 and g(b2) = a2.

The composite function gf: A --> B --> A is

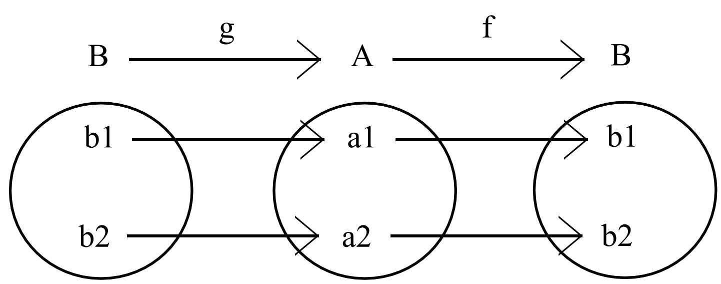


which is equal to

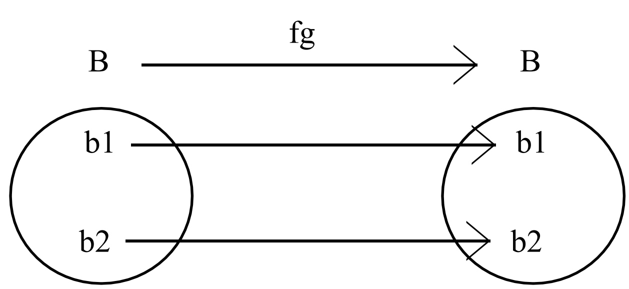


Looking above we see gf: A --> A = 1_A, i.e. 1_A(a1) = a1 and 1_A(a2) = a2.

The composite function fg: B --> A --> B is



which is equal to



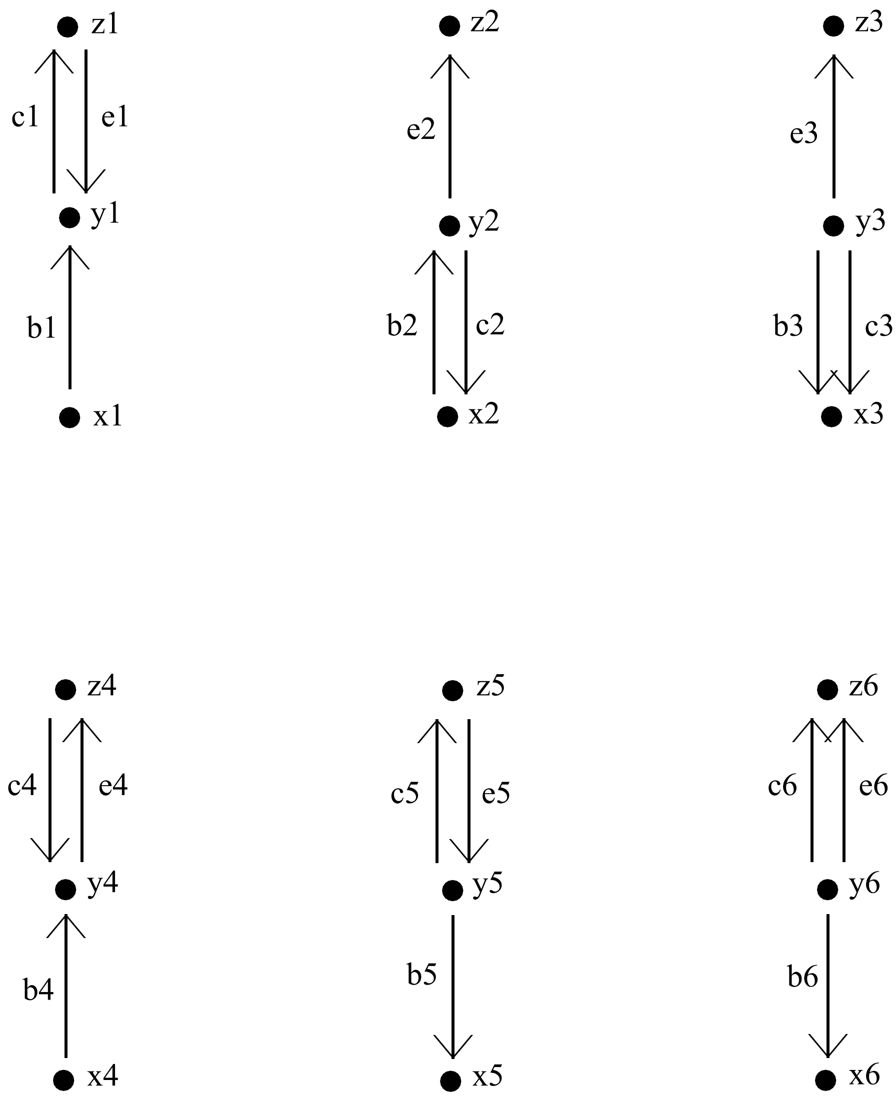
Looking above, one more time, we see that fg: B --> B = 1_B, i.e. 1_B(b1) = b1 and 1_B(b2) = b2.

So we say A and B are isomorphic; are of the same size without even counting the number of elements of either set A or B. I guess this is what it means to participate in the practice of plain-sight, of stating the obvious.

To be continued…

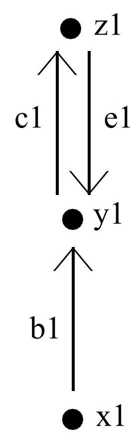
Exercise 6 (Conceptual Mathematics, page 159)

Let's complete Exercise 6; we have a long ways to separating in the category of graphs (Conceptual Mathematics, page 215).

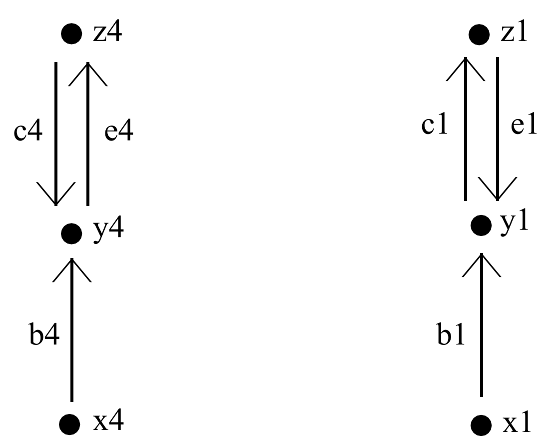


Each one of the above six graphs is isomorphic to exactly one of the other five graphs.

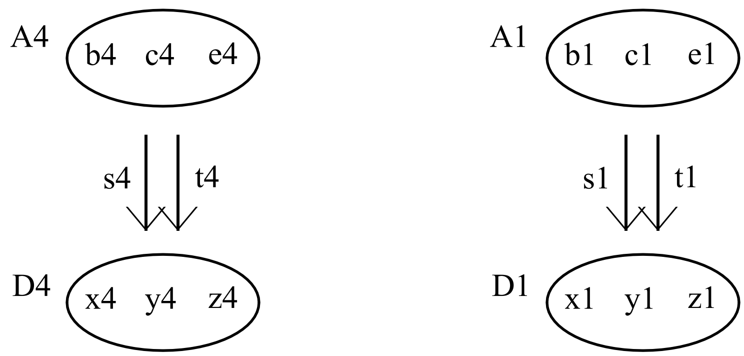
Let's start with graph 1.



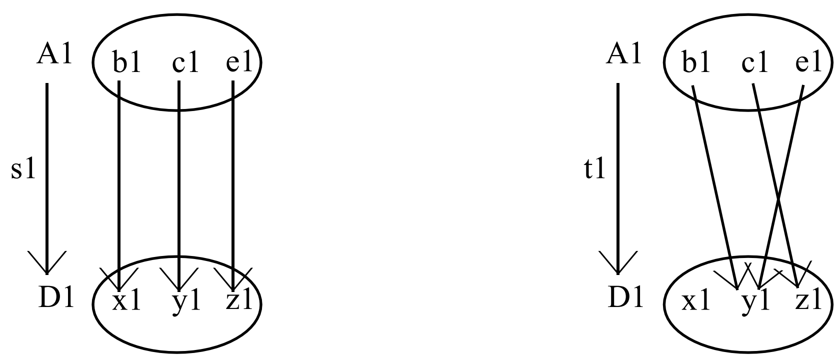
Looking at the other five graphs, it appears as though graph 4 is like graph 1. Let's place them next to one another.



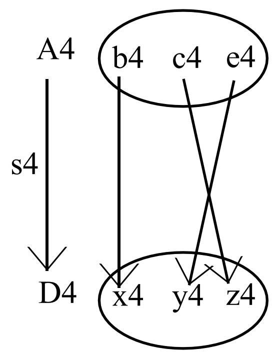
Let's now see if there is an isomorphism between the above two graphs depicted below.

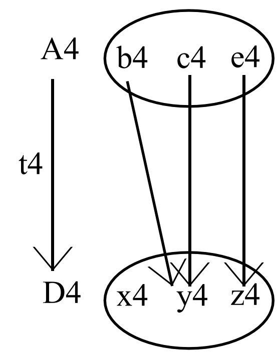


Let's first note that s_1: A1 --> D1 and t_1: A1 --> D1 are given as shown below.

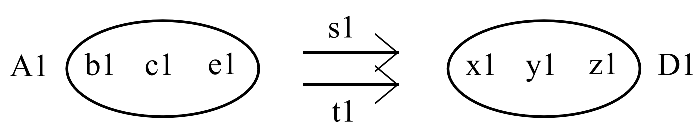


So are s_4: A4 --> D4 and t_4: A4 --> D4 as shown below.

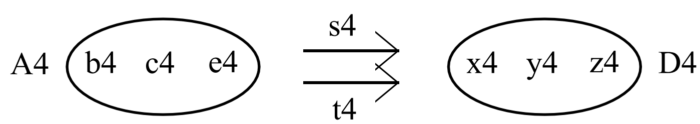




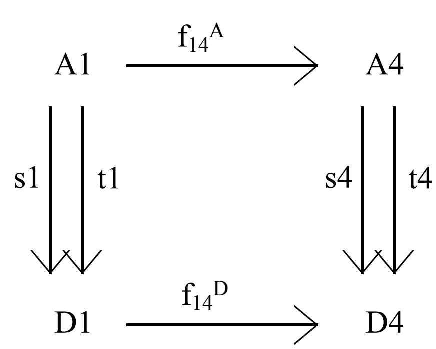
Now to show that the graph



is isomorphic to the graph



we need to find an isomorphism f_{14}: (s1, t1) --> (s4, t4)

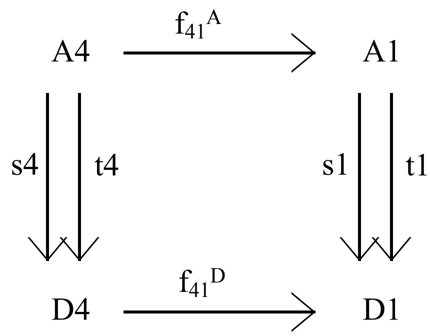


In order for $f_{14} = <f_{14}^A, f_{14}^D>$ to be an isomorphism, first, it has to be a map in the category of graphs satisfying $s_4 f_{14}^A = f_{14}^D s_1$ and $t_4 f_{14}^A = f_{14}^D t_1$

(don't we love subscripts and superscripts; oops, no venting)

Next up, in order for the map $f_{14} = <f_{14}^A, f_{14}^D>$ to be an isomorphism, we need a map

$f_{41} = <f_{41}^A, f_{41}^D>$

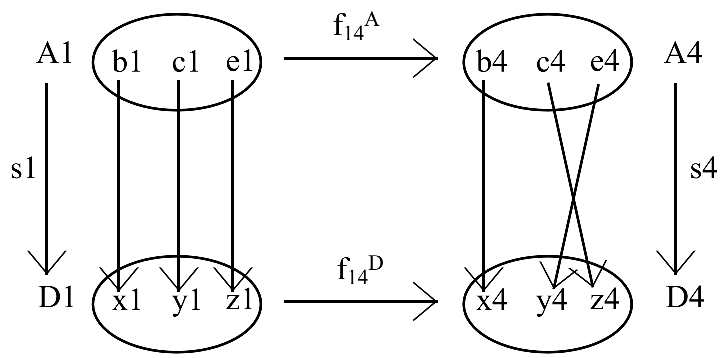


which, by virtue of being a map, satisfies $s_1 f_{41}^A = f_{41}^D s_4$ and $t_1 f_{41}^A = f_{41}^D t_4$ and along with $f_{14} = <f_{14}^A, f_{14}^D>$ satisfying

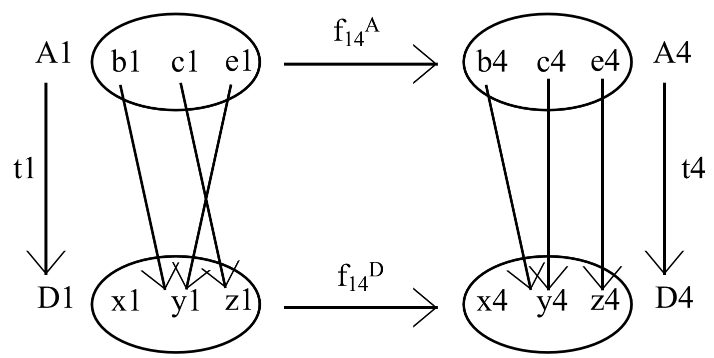
$f_{41}^A f_{14}^A = 1_{A1}$ and $f_{41}^D f_{14}^D = 1_{D1}$

$f_{14}^A f_{41}^A = 1_{A4}$ and $f_{14}^D f_{41}^D = 1_{D4}$

Let's start with $f_{14} = <f_{14}^A, f_{14}^D>$ depicted as



and



satisfying $s_4f_{14}^A = f_{14}^Ds_1$ and $t_4f_{14}^A = f_{14}^Dt_1$

which is short-hand for a more verbose statement saying for $f_{14} = <f_{14}^A, f_{14}^D>$ to be a map in the category of graphs, the functions f_{14}^A and f_{14}^D should be such that if the function f_{14}^A: A1 --> A4 assigns an arrow in the codomain set A4 = {b4, c4, e4} to an arrow in the domain set A1 = {b1, c1, e1}, then the function f_{14}^D: D1 --> D4 must assign a dot in the codomain set D4 = {x4, y4, z4} to each one of the dots in the domain set D1 = {x1, y1, z1} in such a way so as to preserve the source, target relations of arrows in the domain graph (A1, D1) in the codomain graph (A4, D4) (pardon me for being cryptic here; gettin lazy).

Once we find a pair of functions $<f_{14}^{A}, f_{14}^{D}>$ satisfying

$s_4 f_{14}^{A} = f_{14}^{D} s_1$ and $t_4 f_{14}^{A} = f_{14}^{D} t_1$

we, then, have to find another pair of functions $<f_{41}^{A}, f_{41}^{D}>$ satisfying

$s_1 f_{41}^{A} = f_{41}^{D} s_4$ and $t_1 f_{41}^{A} = f_{41}^{D} t_4$

Then we have to see if the maps f_{14} and f_{41} are inverses of one another satisfying

$f_{41}^{A} f_{14}^{A} = 1_{A1}$ and $f_{41}^{D} f_{14}^{D} = 1_{D1}$

$f_{14}^{A} f_{41}^{A} = 1_{A4}$ and $f_{14}^{D} f_{41}^{D} = 1_{D4}$

Once we have an isomorphism between graph 1 and graph 4, that is once we have seen that graph 1 is isomorphic to graph 4 (we also have seen that graph 4 is isomorphic to graph 1, which is reminiscent of saying *saying A = B is same as saying B = A*; here it may be of some interest to note cases wherein, going by some metric, for example, dog may be similar to animal without necessarily asserting that animal is similar to dog; think of arrow vs. loop also), we have to show that graph 1 is not isomorphic to the other four graphs (graph 2, graph 3, graph 5, and graph 6); while we are at it we might as well show that graph 4 is also not isomorphic to graph 2, graph 3, graph 5, and graph 6, which subliminally reads like we are too comfy in here and are somewhat little less than enthusiastic to face the unfamiliar universal properties of the familiar addition (1 + 1 = 2) as if afraid of something short of an excursion from

1 + 1 = 2

to

1 apple + 1 orange = 2 fruits

in thought.