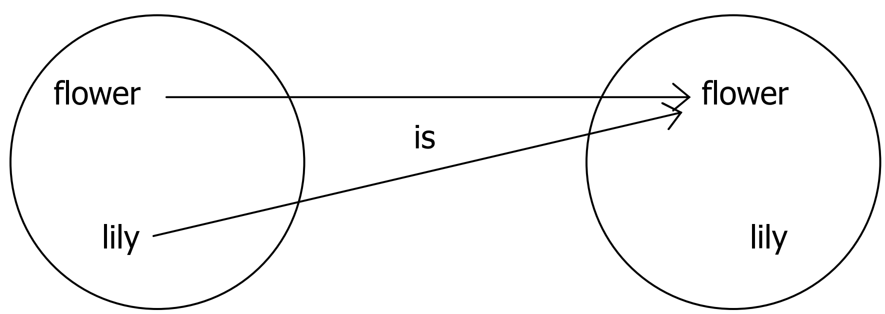
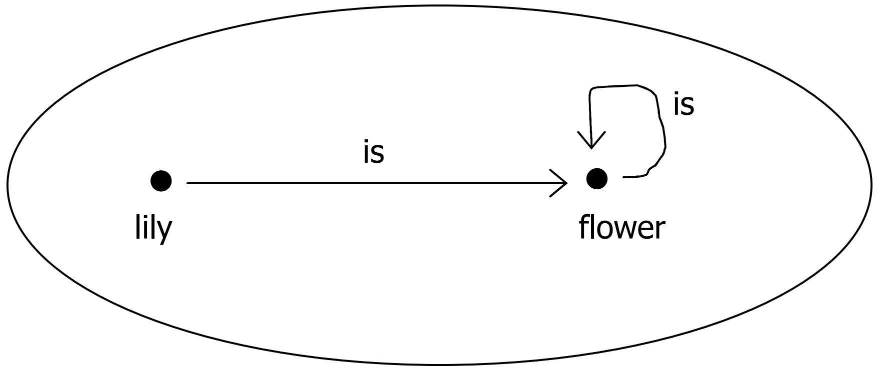
Categorical Perception and Part-Whole Relations

Let's begin with function f: A → B, where A is a set called domain and B is a set called codomain. It is very important to note that both the domain and the codomain are integral to the function just as the two end-points of a line segment are integral to the line. We may also want to note that the domain and codomain need not be different sets. When the domain and codomain are the same set A, then the function f: A → A is called endofunction (also known as endomap).

Let's look at a simple endomap, f: A → A, with A = {flower, lily}



In the above internal diagram the arrows can be interpreted as 'is', so that we can read the diagram as 'flower is flower' and 'lily is flower'. We can also draw the above internal diagram, in view of the fact that both domain and codomain are one and the same set, as follows:

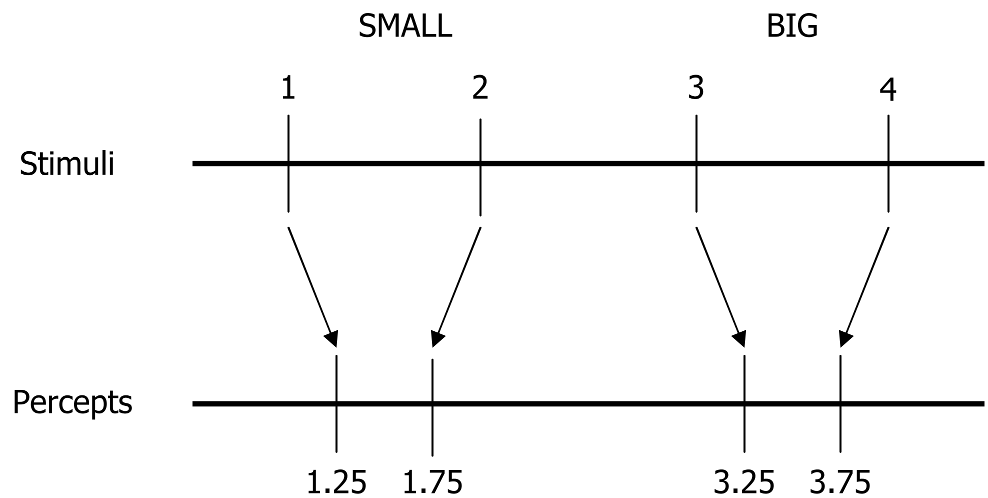


where dots stand for ‘lily’ and ‘flower’, and arrows denote ‘is’.

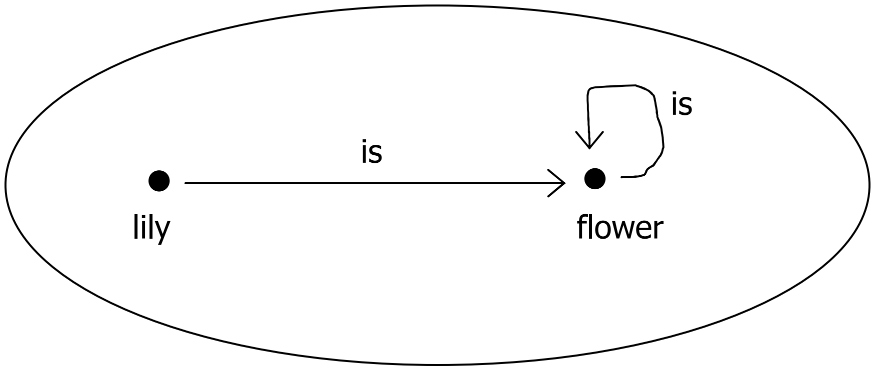
Switching gears slightly, the above diagram happens to be a model of perception, wherein particulars such as ‘lily’ are perceived as exemplars of the corresponding category such as ‘flower’. Let’s make the model little bit more concrete by considering a numerical example. Consider the concrete particulars or physical stimuli, S = {1, 2, 3, 4}, where the elements of S can be thought of as intensities of light, or heights of people, etc. Now consider two abstract generals, say, ‘SMALL’ and ‘BIG’, which we can treat as the names of two categories. To each of these abstract generals there corresponds a concrete general, which can be thought of as the prototype of the corresponding category. The prototypes corresponding to the categories ‘SMALL’ and ‘BIG’ are 1.5 and 3.5, respectively (which are simply the averages of concrete particulars falling under the corresponding categories). Now, let’s depict in tabular form the process of going from physical stimulus to perceptual experience, as follows:

Stimuli (concrete particulars)	1	2	3	4
Categories (abstract generals)	SMALL		BIG	
Prototypes (concrete generals)	1.5 (1 + 2)/2		3.5 (3 + 4)/2	
Percepts	1.25 (1 + 1.5)/2	1.75 (2 + 1.5)/2	3.25 (3 + 3.5)/2	3.75 (4 + 3.5)/2

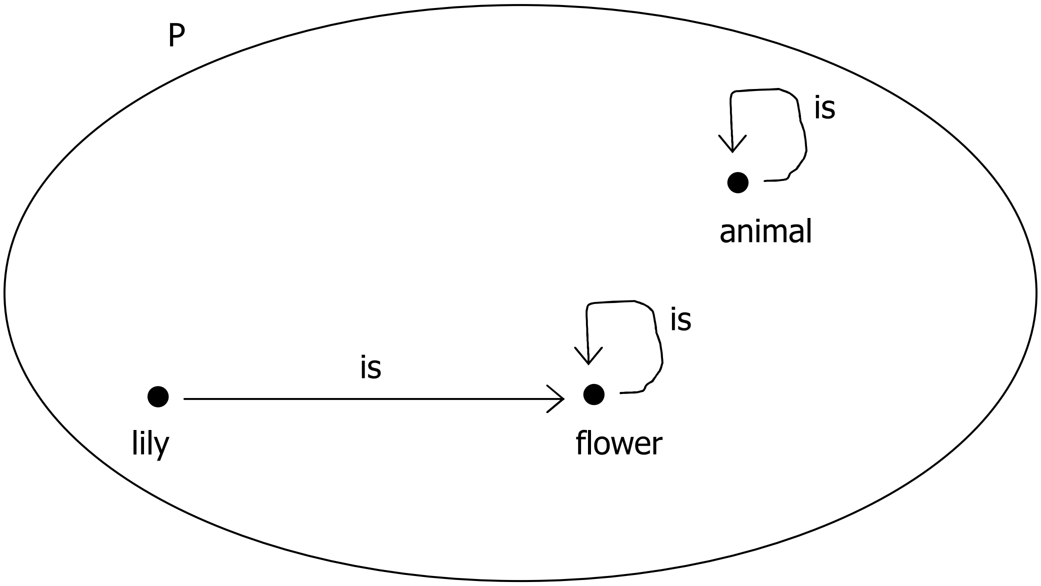
To see more clearly the transformation from physical stimuli to percepts:



From the above stimulus-percept transformation we can see why all the bananas look alike and unlike apples. Summing up, we perceive particular physical stimuli in terms of mental categories (such as SMALL and BIG) to which they belong. All of the involved processes can be abstracted in the following simplified internal diagram (reproduced from earlier):

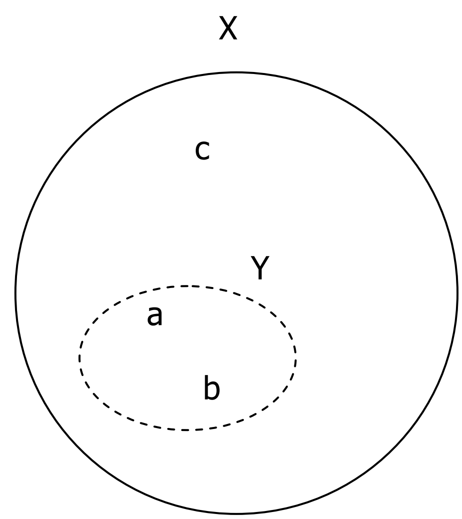


Now to make things little bit more interesting, consider the following perceptual universe:



Let's try to find out its logical structure. Before we go any further, let's set our words straight: 'what do we mean by 'logic'?' We can think of logic as the algebra of parts or even more plainly as part-whole relations.

Let's begin simple. Consider a set X = {a, b, c}, and a part of X, say, Y = {a, b} as depicted below:



Now if we ask 'is, say, 'a' in Y?'

We get the answer: YES

If we ask 'is 'c' in Y?'

We get the answer: NO

These are the only two possible relations that a structure-less element such as 'a' can have with respect to a discrete set such as Y. This is our familiar Boolean logic with its truth-value object of two elements:

Ω_B = {true, false}.

If we note

true = not (false)

false = not (true)

we find that

not (not (A)) = A

where A is an element of the truth-value object Ω_B.

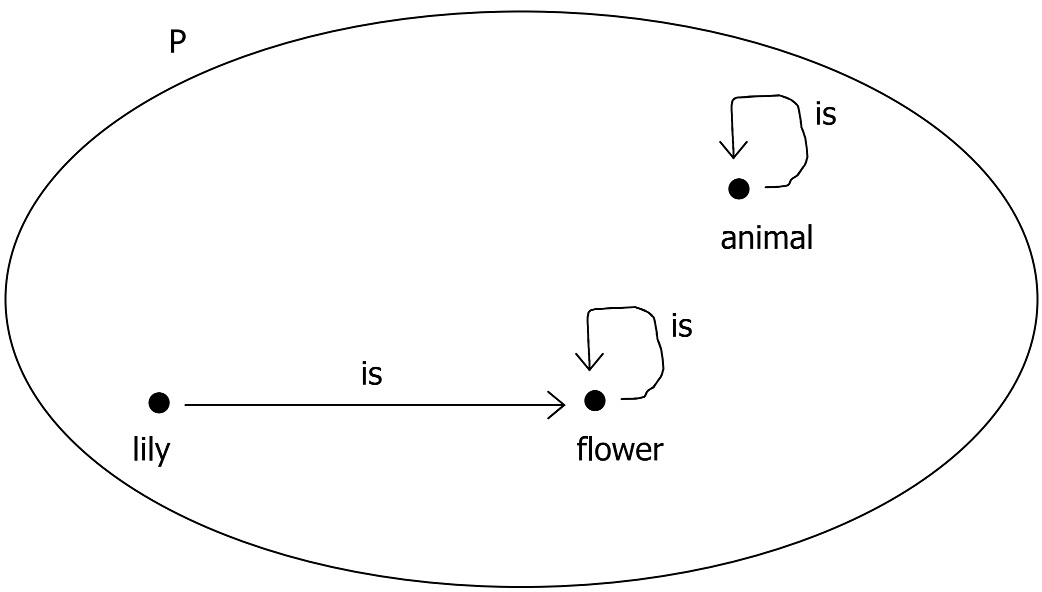
For example,

not (not (true)) = not (false) = true

similarly,

not (not (false)) = not (true) = false

Now let's go back to our perceptual universe P (shown below) and try to characterize its logical structure.



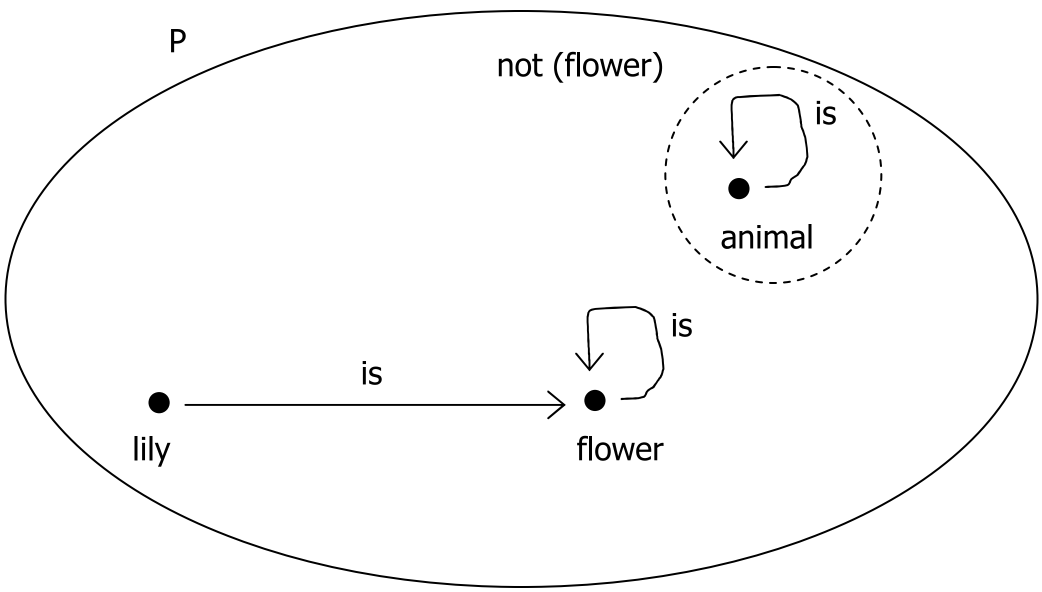
We have just noted that in Boolean algebra

not (not (A)) = A

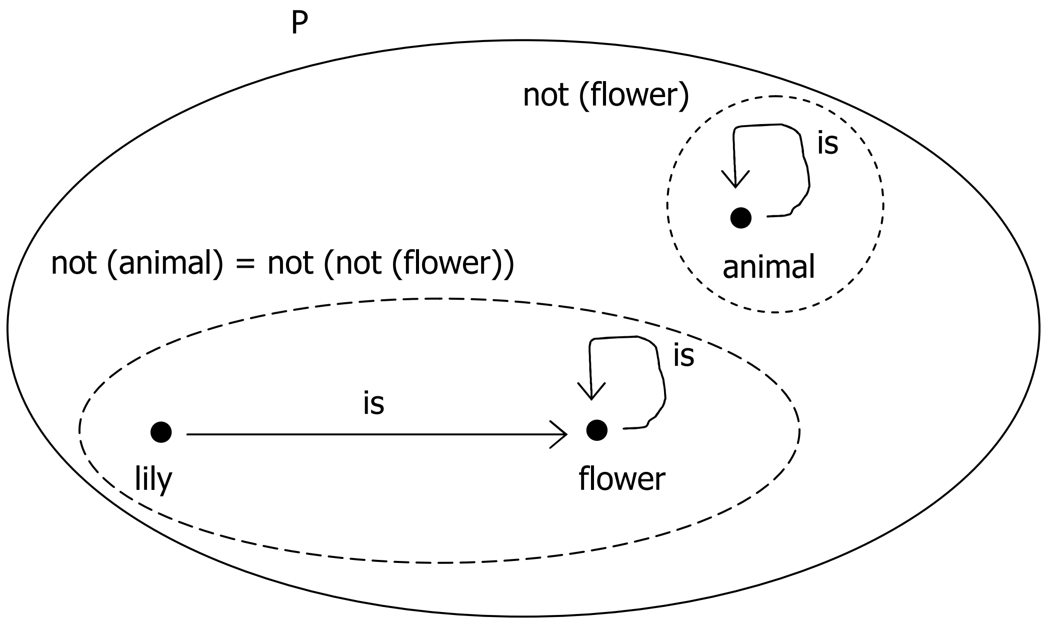
Now let's see if this identity or the law of excluded-middle (either YES or NO; nothing in between) holds water in our perceptual universe P.

Let's begin with 'flower' and ask 'what is 'not (flower)'?' in our perceptual universe P. Looking at the above diagram

not (flower) = animal



Now if we ask: ‘what is ‘not (animal)’?’ we find that ‘not (animal)’ or ‘not (not (flower))’ is not just ‘flower’, but also ‘lily’ as illustrated in the following internal diagram:



Thus we find that

not (not (flower)) ≠ flower

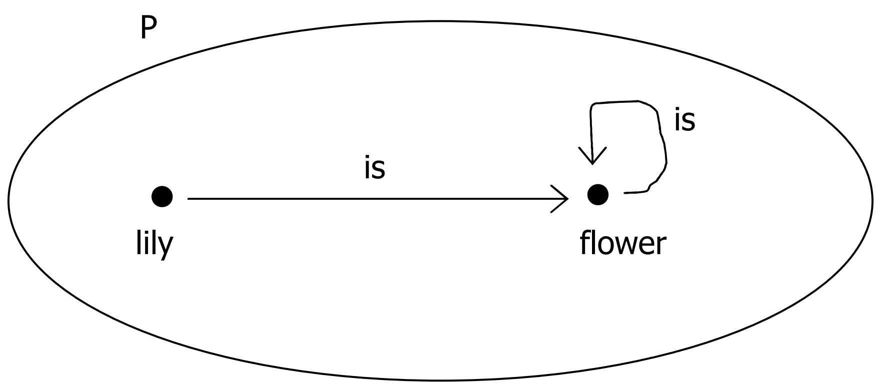
The above case of not (not (A)) ≠ A is just an illustration of Heyting algebra.

Now let’s ask, ‘how about truth-value object?’

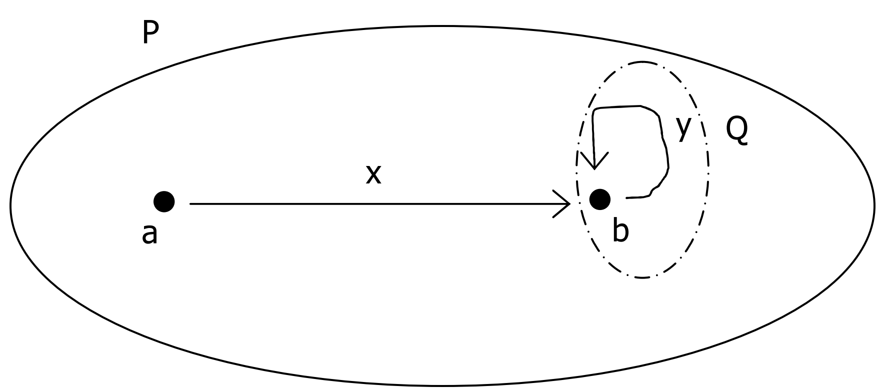
In the case of structure-less elements, a truth-value object:

Ω_B = {true, false}

of two elements suffices to capture all possible relations a part may have with respect to a whole. Since we are dealing with more structured objects (arrows with a source and a target) in the case of our perceptual universe, we can guess that the truth-value object in all likelihood needs more than 2 elements to capture the part-whole relations in the perceptual universe P. Let's redraw a simple perceptual universe as below:



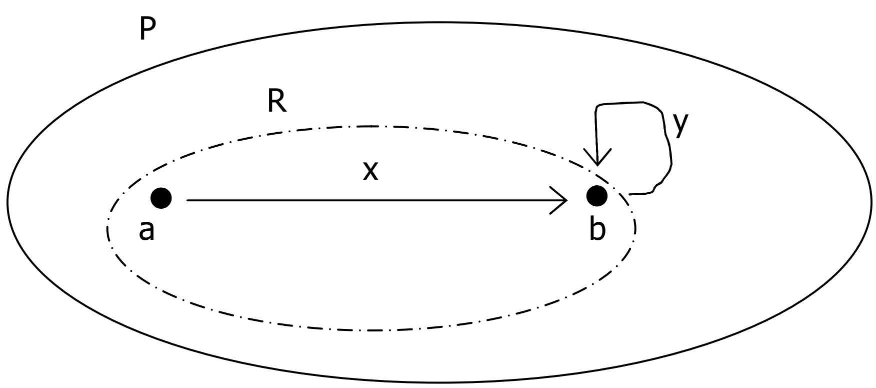
In our universe P, there are dots, and then there are arrows with a source and a target. First let's consider the case of dots in a simplified (in terms of labeling) version of P:



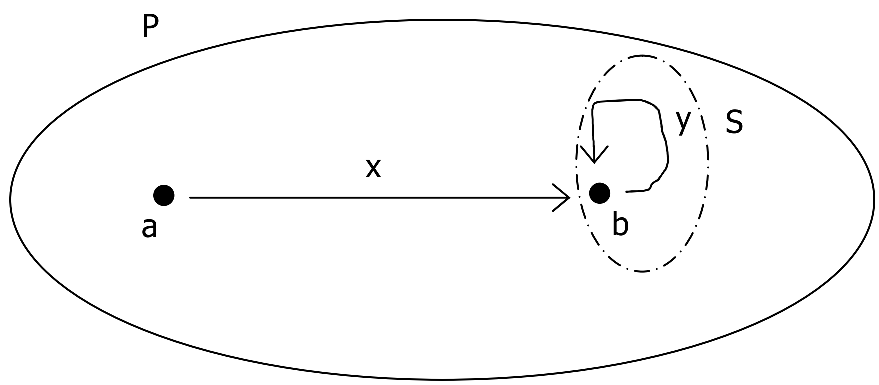
Is dot ‘b’ in ‘Q’? YES

Is dot ‘a’ in ‘Q’? NO

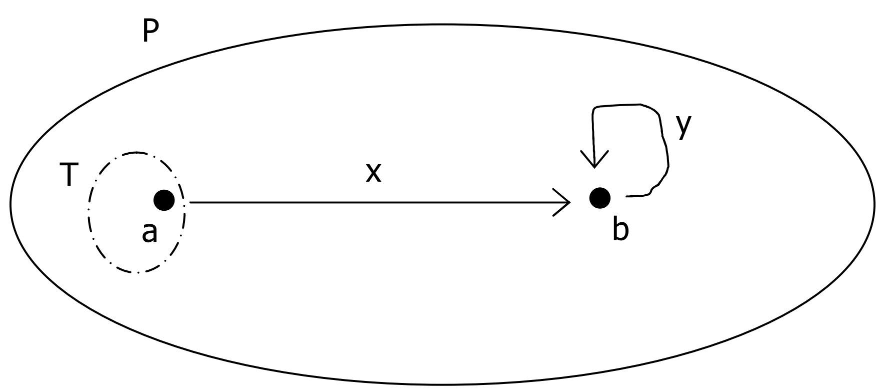
So in the case of dots, the part-whole relations can be captured with two elements: YES, NO. Now let’s look at the case of arrows:



1. The arrow and its source and its target are in part R; e.g. arrow x, its source a, and its target b are in R.
2. The arrow is not in R, but its source and its target are in R; e.g. arrow y is not in R, but its source b and its target b are in R.

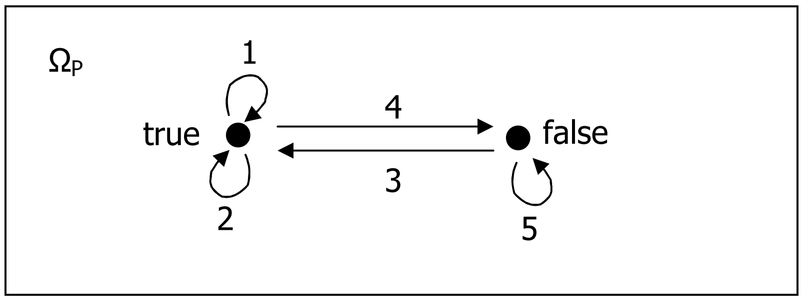


3. The arrow and its source are not in S, but its target is in S; e.g. arrow x and its source a are not in S, but its target b is in S.



4. The arrow and its target are not in part T, but its source is in T; e.g. arrow x and its target b are not in T, but its source a is in T.
5. The arrow, its source, and its target are not in T; e.g. arrow y, its source b, and its target b are not in T.

Summing up, the truth-value object of our perceptual universe P has a total of 7 elements: 2 for dots, and 5 for arrows, which can graphed as follows:



Question: What would be the truth-value object of percept P (the following diagram) taken as a unitary whole (the way we took the arrow along with its source and target as a unitary whole)?